Palindromic structures in lexicographically sorted matrices

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**Introduction**

I am not a mathematician so for those who are this will probably come across as the writings of a mad man. Also, apologies in advance for the many errors I am probably about to make.

A quick background. A while ago I wanted to find the best performing C# program to produce all the permutations of a given tuple which has no repeating elements in lexicographical order. Having looked at some solutions on Stack Overflow I wondered if there was a general algorithm that would generate what I wanted in any base . Although I haven’t found that yet after playing around with the idea on a spreadsheet, I've made the following observations and have come up with some conjectures for lexicographically sorted matrices of . Note that some of the information in this document may already be trivial common knowledge but has been included for completeness.

In addition to the information provided here further examples can be found in the Excel document  
“Palindromic structures in lexicographically sorted matrices.xlsx”

Table of Contents

[1 Definitions 3](#_Toc85847276)

[2 Initial setup 4](#_Toc85847277)

[3 Conjectures 5](#_Toc85847278)

[3.1 Sorting L-Matrix rows is the same as reordering columns 5](#_Toc85847279)

[3.2 L-Matrix row intervals are palindromic irrespective of the sort column 6](#_Toc85847280)

[3.3 An equidistant pair of sequences remain equidistant when rotated 7](#_Toc85847281)

[3.4 An equidistant pair of sequences remain paired irrespective of the sort order 8](#_Toc85847282)

[3.5 L-Matrices consist of n paired palindromic and n paired self-palindromic sets 9](#_Toc85847283)

[3.6 All permutations of a tuple is a subset of an L-Matrix forming a palindromic sequence 12](#_Toc85847284)

[3.7 Any lexicographically sorted set of permutations of base satisfies conjectures 1 - 5 13](#_Toc85847285)

[4 Other observations 16](#_Toc85847286)

[4.1 The sort order is not allways a reversible operation 16](#_Toc85847287)

[4.2 Each column total in an L-Matrix is 19](#_Toc85847288)

[4.3 Palindromic and self-palindromic n value is a factor of the L-matrix value 21](#_Toc85847289)

[4.4 A self-palindromic set’s sequence has the same mean as 22](#_Toc85847290)

[Computational proof of conjecture 4.5 for L-Matrices and permutations 23](#_Toc85847291)

[Appendix A – Computational results for L-Matrices 24](#_Toc85847292)

[Appendix B – Computational results for permutations only 42](#_Toc85847293)

# Definitions

**Palindromic sequence**

A sequence of integers that is palindromic, e.g. (4, -5, 96, -5, 4). Note, this should not be confused with a set of palindromic numbers, e.g. (131, 424, 131).

**L-Matrix**

A lexicographically sorted matrix of . (See initial setup)

**Median**

For a matrix with an odd number of rows the centre row and for a matrix with an even number of rows the point between the two most centre rows

**n paired palindromic sets** - two sets each containing all possible rotations of a sequence that together are palindromically isomorphic about the median of the matrix with **n** denoting the number of matching pairs of rotations between the two sets. Combined the pairings form a palindromic sequence.

**n paired self-palindromic set** - a set containing all possible rotations of a sequence in which each rotation is palindromically isomorphic about the median of the matrix to another rotation within the same set with **n** denoting the number of matching pairs of rotations within the set. Combined the pairings form a palindromic sequence.

**Sorted y** - the after the matrix is initially sorted smallest to largest according to column the matrix is then sorted according to any column **y** smallest to largest or largest to smallest.

# Initial setup

For

Create a sequence representation of each digit of with leading zeroes such that the number of elements in the sequence is *n*.

Create a matrix . Arrange the columns such that they contain separately each of the digits from each sequence of with column 0 representing and column *n* representing .

Add 2 columns to the left of column . The right column is the value . The left column is the deviation of from the median for a matrix sorted . Where the median is not an integer the column values are rounded to the next integer. Both columns are converted to base 10 for convenience.

Arrange the rows from = 0 in row 0 through to = *bn* -1 in row *bn*.

Example 3.1

The initial state of the matrix is to be sorted *.*

The following conjectures only holds where the matrix is lexicographically sorted to one of the columns.

# Conjectures

## Sorting L-Matrix rows is the same as reordering columns

When the rows in an L-Matrix are resorted according to another columnthe result is the same as reordering the columns.

This is in effect synchronously redefining the place holders of all the numbers.

Example 4.1.1

Given an L-Matrix, , the left-hand matrix is initially sorted  then sorted to create the matrix to the right. This is the same as reordering the columns

## L-Matrix row intervals are palindromic irrespective of the sort column

The intervals between consecutive values of  forms a palindromic sequence irrespective of the sort column or whether the matrix is sorted smallest to largest or largest to smallest.

Example 4.2.1

Given an L-Matrix, , the right-hand matrix is sorted  and the intervals between consecutive values of produces the palindromic sequence

(1, 1, 7, 1, 1, 7, 1, 1, -17, 1, 1, 7, 1, 1, 7, 1, 1, -17, 1, 1, 7, 1, 1, 7, 1, 1)

This also holds true for a matrix sorted to *bn* as with the matrix on the left but may be overlooked at first glance as the intervals are all 1’s.

(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)

## An equidistant pair of sequences remain equidistant when rotated

Let , and be a pair of sequences that are together equidistant from the median of an L-Matrix. When each sequence is rotated in the same direction by the same amount the two resulting sequences are also equidistant about the median.

Example 4.3.1

Given an L-Matrix, , there exists a paired sequence on Offset -12 (0, 0, 1) and   
12 (2, 2, 1). When rotated right by one position the transformed sequences are located on   
Offset -4 (1, 0, 0) and 4 (1, 2, 2) respectively. When rotated again the transformed sequences are on Offset -10 (0, 1, 0) and 10 (2, 1, 2). As can be seen for all transformations the pairs are equidistant from the median.

## An equidistant pair of sequences remain paired irrespective of the sort order

The pairing of an equidistant pair of sequences and therefore the equivalent value of  remains paired irrespective of the sort column or whether the matrix is sorted smallest to largest or largest to smallest.

Example 4.4.1

Given an L-Matrix, , the right-hand matrix is sorted *b1*. As can be seen the paired sequences on offsets -10 and 10 have swapped with the sequences on offsets -4 and 4 respectively and as such the pairings have not changed.

## L-Matrices consist of n paired palindromic and n paired self-palindromic sets

L-Matrices are comprised entirely of either n paired palindromic sets only or a combination of   
n paired palindromic sets and n paired self-palindromic sets.

Note: This has been confirmed computationally for   
L-Matrices of

Example 4.5.1

Given an L-Matrix, , there exists a set on offsets -12, -10 and -4 of all rotations of the sequence (0, 0, 1), highlighted orange, and an isomorphic set on offsets 4, 10 and 12 of all rotations of the sequence (1, 2, 2), highlighted green. Combined the intervals form the palindromic sequence  
(2, 6, 8, 6, 2)

Example 4.5.2

Some **n** paired palindromic sets are asymmetrically distributed around the median. As shown below in the same matrix as above there exists a set on offsets -11, -7, 5 of all rotations of the sequence (0, 0, 2), highlighted orange, and an isomorphic set on offsets -5, 7, 11 of all rotations of the sequence (0, 2, 2), highlighted green. Combined the intervals form the palindromic sequence   
(4, 2, 10, 2, 4).

Note: If an asymmetrically distributed palindromic set is resorted such that each subset a of rotated sequence is separated the intervals still form a palindromic sequence.

Example 4.5.3

The following is an example of n paired self-palindromic set comprising all the rotations of the sequence (0, 0, 1, 1). The palindromic sequence here is (3, 2, 3)

## All permutations of a tuple is a subset of an L-Matrix forming a palindromic sequence

Given a tuple the lexicographically ordered set of all the permutations of will be a palindromically distributed subset about the median of an L-matrix of base .

Example 4.6.1

In this example there exists a set of all the permutations of the tuple (0, 1, 2) highlighted green. Together the intervals between each permutation form the palindromic sequence (2, 4, 4, 4, 2).

## Any lexicographically sorted set of permutations of base satisfies conjectures 1 - 5

Given a tuple the lexicographically ordered set P of all the permutations of will satisfy conjectures 4.1 through 4.5.

Example 4.7.1

Let set be the lexicographically sorted set of all permutations of the tuple (0, 1, 2) as indicated by columns  . The left-hand matrix is sorted and the right-hand matrix has been   
sorted . In this example

Conjecture 4.1 has been satisfied as the columns for set have been reordered  
 in the right-hand matrix.

Conjecture 4.2 is satisfied as the row intervals in both cases form a palindromic sequence. For the left-hand matrix it is (2, 4, 4, 4, 2) and for the right-hand matrix it is (8, -14, 16, -14, 8)

Conjecture 4.3 is satisfied as when the equidistant pair of sequences on Offset -8 and 8 are each rotated right 1 position, they are relocated to Offset 2 and -2 respectively as seen in the left-hand matrix thus remaining equidistant.

Conjecture 4.4 is satisfied as the paired sequences on Offset -8 and 8 in the left-hand matrix are relocated to Offset 6 and –6 respectively in the resorted right-hand matrix thus remaining paired.

Conjecture 4.5 is satisfied as in the left-hand matrix there exists a set 𝑆𝑎 on rows -8, 2 and 6 of all rotations of the sequence (0, 1, 2), highlighted orange, and an isomorphic set 𝑆𝑏 on rows -6, -2 and 6 of all rotations of the sequence (0, 2, 1), highlighted green. Combined the intervals form the palindromic sequence (2, 4, 4, 4, 2). When the matrix is resorted such that each subset of rotated sequences is separated the intervals still from a palindromic sequence (10, 4, -12, 4, 10).

Example 4.7.2

The following is another demonstration of conjecture 4.5. Let set be the lexicographically sorted set of all permutations of the tuple (0, 1, 2, 3) as indicated by columns .

Conjecture 4.5 is satisfied as set consists entirely of a combination of n paired palindromic and   
n paired self-palindromic sets.

There are 2 x 4 paired palindromic sets and 2 x 2 paired self-palindromic sets which are as shown on the next page.

Paired palindromic set on Offset (-101, -20, 50, 71) and (-71, -50, 20, 101) with values of   
(27, 108, 177, 198) and (57, 78, 147, 228) respectively. Combined the intervals form the palindromic sequence (30, 21, 30, 40, 30, 21, 30)

Paired palindromic set on Offset (-89, -14, 29, 74) and (-74, -29, 14, 89) with values of   
(39, 114, 156, 201) and (54, 99, 141, 216) respectively. Combined the intervals form the palindromic sequence (15, 45, 15, 28, 15, 45, 15)

Self-paired palindromic set on (-98, -8, 8, 98) with values of (30, 120, 135, 225) and combined the intervals form the palindromic sequence is (90, 16, 90)

Self-paired palindromic set on (-83, -53, 53, 83) with values of (45, 75, 180, 210) and combined the intervals form the palindromic sequence is (30, 106, 30)

# Other observations

The following are a list of observations I have made while exploring this document’s topic. They are included here mainly as a reminder to myself of avenues of exploration but may be also of interest to the reader.

## The sort order is not allways a reversible operation

When rows of an L-Matrix are sorted according to another columnsorting the rows by the previous column may not always return the matrix back to the previous state.

Example 5.1.1

Given an L-Matrix, , when the left-hand matrix is sorted the columns will be reordered .When resorted back to column however instead of  
 the result is

Example 5.1.2

Given the same matrix as above when the left-hand matrix is sorted column the columns will be reordered and when resorted back to the they are reordered this time restoring the matrix back to its original state.

As with an L-Matrix when the rows of a set of the permutations of set are sorted according to another columnsorting the rows by the previous column may not always return the set back to the previous state.

Example 5.1.3

Given the set for when the left-hand set is sorted the columns will be reordered .When resorted back to column however instead of  
 the result is

Example 5.1.4

Given the same set as above when the left-hand set is sorted column the columns will be reordered and when resorted back to the they are reordered this time restoring the set back to its original state.

## Each column total in an L-Matrix is

The total of any column of an L-Matrix is .

Example 5.2.1

Given an L-Matrix,

Example 5.2.2

Given an L-Matrix,

## Palindromic and self-palindromic n value is a factor of the L-matrix value

From inspection of the results in Appendix A, the n value of n paired palindromic and   
n paired self-palindromic sets is a factor of the L-matrix value. For n paired self-palindromic sets this excludes the L-matrix value itself.

Example 5.3.1

Given an L-Matrix, , the n values for the paired palindromic sets are 1, 2, 4, 8 and the  
n values for the paired self-palindromic sets are 1, 2, 4.

## A self-palindromic set’s sequence has the same mean as

For a set to be self-palindromic, the L-matrix value must be even and the mean of the set’s sequence must be equal to the mean of the set (0 ... b-1) but not all sets that satisfy these requirements are self-palindromic.

Example 5.4.1

Given an L-Matrix, , the set = (0, 1, 2) and the mean is 1. The sequences that also have a mean of 1 are (0, 0, 2, 2), (0, 1, 1, 2), (0, 1, 2, 1), (0, 2, 0, 2) and (0, 2, 1, 1) of which  
(0, 0, 2, 2), (0, 1, 2, 1) and (0, 2, 0, 2) are self-palindromic.

Note: the matrix for this example is too large to include here but can be found in the Excel workbook  
“Palindromic structures in lexicographically sorted matrices.xlsx”.

**Computational proof of conjecture 4.5 for L-Matrices and permutations**

The following is the method used for computational proof of conjecture 4.5 for all L-Matrices of and the lexicographically sorted set of all permutations for tuple for .

Results for L-Matrices are shown in Appendix A and permutations only in Appendix B.

1. Start
2. Let
3. Let
4. For
5. Let be the left zero padded sequence of the digits of
6. Let be a set containing all the rotated sequences of including
7. If we’re checking permutations only and contains a sequence that does not have distinct elements then continue with the next value of
8. If contains a sequence with an equivalent value less than then was previously checked so continue with the next value of
9. Let ’ be a copy of
10. For each sequence in ’ let be the equivalent value of
11. If ’ contains another sequence with an equivalent value of ,   
    remove both and from ’
12. If ’ is now empty it was self-palindromic so add it to the results and continue with the next value of
13. If ’ is not empty but has less elements than then it was neither completely self-palindromic or completely isomorphic with another set so the conjecture has failed and we can stop checking
14. Let Set be the left zero padded sequence of digits of
15. Remove from
16. For the number of sequences remaining in
17. Rotate right 1 element
18. Let be the equivalent value of
19. If does not contain a sequence with an equivalent value of then the conjecture has failed and we can stop checking
20. Else add to the results and continue with the next value of
21. End

**Appendix A – Computational results for L-Matrices**

Checking palindromicity for Base = 2 to 10 and n = 2 to 10

Base = 2, n = 2: Matrix is palindromic

Maximum matrix number = 3

1 paired palindromic sets = 1

1 paired self-palindromic sets = 1

Total pairs = 2

Total paired sets = 2

Total sets = 3

Base = 2, n = 3: Matrix is palindromic

Maximum matrix number = 7

1 paired palindromic sets = 1

3 paired palindromic sets = 1

Total pairs = 4

Total paired sets = 2

Total sets = 4

Base = 2, n = 4: Matrix is palindromic

Maximum matrix number = 15

1 paired palindromic sets = 1

4 paired palindromic sets = 1

1 paired self-palindromic sets = 1

2 paired self-palindromic sets = 1

Total pairs = 8

Total paired sets = 4

Total sets = 6

Base = 2, n = 5: Matrix is palindromic

Maximum matrix number = 31

1 paired palindromic sets = 1

5 paired palindromic sets = 3

Total pairs = 16

Total paired sets = 4

Total sets = 8

Base = 2, n = 6: Matrix is palindromic

Maximum matrix number = 63

1 paired palindromic sets = 1

3 paired palindromic sets = 1

6 paired palindromic sets = 4

1 paired self-palindromic sets = 1

3 paired self-palindromic sets = 1

Total pairs = 32

Total paired sets = 8

Total sets = 14

Base = 2, n = 7: Matrix is palindromic

Maximum matrix number = 127

1 paired palindromic sets = 1

7 paired palindromic sets = 9

Total pairs = 64

Total paired sets = 10

Total sets = 20

Base = 2, n = 8: Matrix is palindromic

Maximum matrix number = 255

1 paired palindromic sets = 1

4 paired palindromic sets = 1

8 paired palindromic sets = 14

1 paired self-palindromic sets = 1

2 paired self-palindromic sets = 1

4 paired self-palindromic sets = 2

Total pairs = 128

Total paired sets = 20

Total sets = 36

Base = 2, n = 9: Matrix is palindromic

Maximum matrix number = 511

1 paired palindromic sets = 1

3 paired palindromic sets = 1

9 paired palindromic sets = 28

Total pairs = 256

Total paired sets = 30

Total sets = 60

Base = 2, n = 10: Matrix is palindromic

Maximum matrix number = 1023

1 paired palindromic sets = 1

5 paired palindromic sets = 3

10 paired palindromic sets = 48

1 paired self-palindromic sets = 1

5 paired self-palindromic sets = 3

Total pairs = 512

Total paired sets = 56

Total sets = 108

Base = 3, n = 2: Matrix is palindromic

Maximum matrix number = 8

Median = 4, median sequence = 1, 1

1 paired palindromic sets = 1

2 paired palindromic sets = 1

1 paired self-palindromic sets = 1

Total pairs = 4

Total paired sets = 3

Total sets = 5

Base = 3, n = 3: Matrix is palindromic

Maximum matrix number = 26

Median = 13, median sequence = 1, 1, 1

1 paired palindromic sets = 1

3 paired palindromic sets = 4

Total pairs = 13

Total paired sets = 5

Total sets = 10

Base = 3, n = 4: Matrix is palindromic

Maximum matrix number = 80

Median = 40, median sequence = 1, 1, 1, 1

1 paired palindromic sets = 1

2 paired palindromic sets = 1

4 paired palindromic sets = 8

1 paired self-palindromic sets = 1

2 paired self-palindromic sets = 2

Total pairs = 40

Total paired sets = 13

Total sets = 23

Base = 3, n = 5: Matrix is palindromic

Maximum matrix number = 242

Median = 121, median sequence = 1, 1, 1, 1, 1

1 paired palindromic sets = 1

5 paired palindromic sets = 24

Total pairs = 121

Total paired sets = 25

Total sets = 50

Base = 3, n = 6: Matrix is palindromic

Maximum matrix number = 728

Median = 364, median sequence = 1, 1, 1, 1, 1, 1

1 paired palindromic sets = 1

2 paired palindromic sets = 1

3 paired palindromic sets = 4

6 paired palindromic sets = 56

1 paired self-palindromic sets = 1

3 paired self-palindromic sets = 4

Total pairs = 364

Total paired sets = 67

Total sets = 129

Base = 3, n = 7: Matrix is palindromic

Maximum matrix number = 2186

Median = 1093, median sequence = 1, 1, 1, 1, 1, 1, 1

1 paired palindromic sets = 1

7 paired palindromic sets = 156

Total pairs = 1,093

Total paired sets = 157

Total sets = 314

Base = 3, n = 8: Matrix is palindromic

Maximum matrix number = 6560

Median = 3280, median sequence = 1, 1, 1, 1, 1, 1, 1, 1

1 paired palindromic sets = 1

2 paired palindromic sets = 1

4 paired palindromic sets = 8

8 paired palindromic sets = 400

1 paired self-palindromic sets = 1

2 paired self-palindromic sets = 2

4 paired self-palindromic sets = 10

Total pairs = 3,280

Total paired sets = 423

Total sets = 833

Base = 3, n = 9: Matrix is palindromic

Maximum matrix number = 19682

Median = 9841, median sequence = 1, 1, 1, 1, 1, 1, 1, 1, 1

1 paired palindromic sets = 1

3 paired palindromic sets = 4

9 paired palindromic sets = 1,092

Total pairs = 9,841

Total paired sets = 1,097

Total sets = 2,194

Base = 3, n = 10: Matrix is palindromic

Maximum matrix number = 59048

Median = 29524, median sequence = 1, 1, 1, 1, 1, 1, 1, 1, 1, 1

1 paired palindromic sets = 1

2 paired palindromic sets = 1

5 paired palindromic sets = 24

10 paired palindromic sets = 2,928

1 paired self-palindromic sets = 1

5 paired self-palindromic sets = 24

Total pairs = 29,524

Total paired sets = 2,979

Total sets = 5,933

Base = 4, n = 2: Matrix is palindromic

Maximum matrix number = 15

1 paired palindromic sets = 2

2 paired palindromic sets = 2

1 paired self-palindromic sets = 2

Total pairs = 8

Total paired sets = 6

Total sets = 10

Base = 4, n = 3: Matrix is palindromic

Maximum matrix number = 63

1 paired palindromic sets = 2

3 paired palindromic sets = 10

Total pairs = 32

Total paired sets = 12

Total sets = 24

Base = 4, n = 4: Matrix is palindromic

Maximum matrix number = 255

1 paired palindromic sets = 2

2 paired palindromic sets = 2

4 paired palindromic sets = 28

1 paired self-palindromic sets = 2

2 paired self-palindromic sets = 4

Total pairs = 128

Total paired sets = 38

Total sets = 70

Base = 4, n = 5: Matrix is palindromic

Maximum matrix number = 1023

1 paired palindromic sets = 2

5 paired palindromic sets = 102

Total pairs = 512

Total paired sets = 104

Total sets = 208

Base = 4, n = 6: Matrix is palindromic

Maximum matrix number = 4095

1 paired palindromic sets = 2

2 paired palindromic sets = 2

3 paired palindromic sets = 10

6 paired palindromic sets = 330

1 paired self-palindromic sets = 2

3 paired self-palindromic sets = 10

Total pairs = 2,048

Total paired sets = 356

Total sets = 700

Base = 4, n = 7: Matrix is palindromic

Maximum matrix number = 16383

1 paired palindromic sets = 2

7 paired palindromic sets = 1,170

Total pairs = 8,192

Total paired sets = 1,172

Total sets = 2,344

Base = 4, n = 8: Matrix is palindromic

Maximum matrix number = 65535

1 paired palindromic sets = 2

2 paired palindromic sets = 2

4 paired palindromic sets = 28

8 paired palindromic sets = 4,064

1 paired self-palindromic sets = 2

2 paired self-palindromic sets = 4

4 paired self-palindromic sets = 32

Total pairs = 32,768

Total paired sets = 4,134

Total sets = 8,230

Base = 4, n = 9: Matrix is palindromic

Maximum matrix number = 262143

1 paired palindromic sets = 2

3 paired palindromic sets = 10

9 paired palindromic sets = 14,560

Total pairs = 131,072

Total paired sets = 14,572

Total sets = 29,144

Base = 4, n = 10: Matrix is palindromic

Maximum matrix number = 1048575

1 paired palindromic sets = 2

2 paired palindromic sets = 2

5 paired palindromic sets = 102

10 paired palindromic sets = 52,326

1 paired self-palindromic sets = 2

5 paired self-palindromic sets = 102

Total pairs = 524,288

Total paired sets = 52,536

Total sets = 104,968

Base = 5, n = 2: Matrix is palindromic

Maximum matrix number = 24

Median = 12, median sequence = 2, 2

1 paired palindromic sets = 2

2 paired palindromic sets = 4

1 paired self-palindromic sets = 2

Total pairs = 12

Total paired sets = 8

Total sets = 14

Base = 5, n = 3: Matrix is palindromic

Maximum matrix number = 124

Median = 62, median sequence = 2, 2, 2

1 paired palindromic sets = 2

3 paired palindromic sets = 20

Total pairs = 62

Total paired sets = 22

Total sets = 44

Base = 5, n = 4: Matrix is palindromic

Maximum matrix number = 624

Median = 312, median sequence = 2, 2, 2, 2

1 paired palindromic sets = 2

2 paired palindromic sets = 4

4 paired palindromic sets = 72

1 paired self-palindromic sets = 2

2 paired self-palindromic sets = 6

Total pairs = 312

Total paired sets = 86

Total sets = 164

Base = 5, n = 5: Matrix is palindromic

Maximum matrix number = 3124

Median = 1562, median sequence = 2, 2, 2, 2, 2

1 paired palindromic sets = 2

5 paired palindromic sets = 312

Total pairs = 1,562

Total paired sets = 314

Total sets = 628

Base = 5, n = 6: Matrix is palindromic

Maximum matrix number = 15624

Median = 7812, median sequence = 2, 2, 2, 2, 2, 2

1 paired palindromic sets = 2

2 paired palindromic sets = 4

3 paired palindromic sets = 20

6 paired palindromic sets = 1,280

1 paired self-palindromic sets = 2

3 paired self-palindromic sets = 20

Total pairs = 7,812

Total paired sets = 1,328

Total sets = 2,634

Base = 5, n = 7: Matrix is palindromic

Maximum matrix number = 78124

Median = 39062, median sequence = 2, 2, 2, 2, 2, 2, 2

1 paired palindromic sets = 2

7 paired palindromic sets = 5,580

Total pairs = 39,062

Total paired sets = 5,582

Total sets = 11,164

Base = 5, n = 8: Matrix is palindromic

Maximum matrix number = 390624

Median = 195312, median sequence = 2, 2, 2, 2, 2, 2, 2, 2

1 paired palindromic sets = 2

2 paired palindromic sets = 4

4 paired palindromic sets = 72

8 paired palindromic sets = 24,336

1 paired self-palindromic sets = 2

2 paired self-palindromic sets = 6

4 paired self-palindromic sets = 78

Total pairs = 195,312

Total paired sets = 24,500

Total sets = 48,914

Base = 5, n = 9: Matrix is palindromic

Maximum matrix number = 1953124

Median = 976562, median sequence = 2, 2, 2, 2, 2, 2, 2, 2, 2

1 paired palindromic sets = 2

3 paired palindromic sets = 20

9 paired palindromic sets = 108,500

Total pairs = 976,562

Total paired sets = 108,522

Total sets = 217,044

Base = 5, n = 10: Matrix is palindromic

Maximum matrix number = 9765624

Median = 4882812, median sequence = 2, 2, 2, 2, 2, 2, 2, 2, 2, 2

1 paired palindromic sets = 2

2 paired palindromic sets = 4

5 paired palindromic sets = 312

10 paired palindromic sets = 487,968

1 paired self-palindromic sets = 2

5 paired self-palindromic sets = 312

Total pairs = 4,882,812

Total paired sets = 488,600

Total sets = 976,886

Base = 6, n = 2: Matrix is palindromic

Maximum matrix number = 35

1 paired palindromic sets = 3

2 paired palindromic sets = 6

1 paired self-palindromic sets = 3

Total pairs = 18

Total paired sets = 12

Total sets = 21

Base = 6, n = 3: Matrix is palindromic

Maximum matrix number = 215

1 paired palindromic sets = 3

3 paired palindromic sets = 35

Total pairs = 108

Total paired sets = 38

Total sets = 76

Base = 6, n = 4: Matrix is palindromic

Maximum matrix number = 1295

1 paired palindromic sets = 3

2 paired palindromic sets = 6

4 paired palindromic sets = 153

1 paired self-palindromic sets = 3

2 paired self-palindromic sets = 9

Total pairs = 648

Total paired sets = 174

Total sets = 336

Base = 6, n = 5: Matrix is palindromic

Maximum matrix number = 7775

1 paired palindromic sets = 3

5 paired palindromic sets = 777

Total pairs = 3,888

Total paired sets = 780

Total sets = 1,560

Base = 6, n = 6: Matrix is palindromic

Maximum matrix number = 46655

1 paired palindromic sets = 3

2 paired palindromic sets = 6

3 paired palindromic sets = 35

6 paired palindromic sets = 3,850

1 paired self-palindromic sets = 3

3 paired self-palindromic sets = 35

Total pairs = 23,328

Total paired sets = 3,932

Total sets = 7,826

Base = 6, n = 7: Matrix is palindromic

Maximum matrix number = 279935

1 paired palindromic sets = 3

7 paired palindromic sets = 19,995

Total pairs = 139,968

Total paired sets = 19,998

Total sets = 39,996

Base = 6, n = 8: Matrix is palindromic

Maximum matrix number = 1679615

1 paired palindromic sets = 3

2 paired palindromic sets = 6

4 paired palindromic sets = 153

8 paired palindromic sets = 104,814

1 paired self-palindromic sets = 3

2 paired self-palindromic sets = 9

4 paired self-palindromic sets = 162

Total pairs = 839,808

Total paired sets = 105,150

Total sets = 210,126

Base = 6, n = 9: Matrix is palindromic

Maximum matrix number = 10077695

1 paired palindromic sets = 3

3 paired palindromic sets = 35

9 paired palindromic sets = 559,860

Total pairs = 5,038,848

Total paired sets = 559,898

Total sets = 1,119,796

Base = 6, n = 10: Matrix is palindromic

Maximum matrix number = 60466175

1 paired palindromic sets = 3

2 paired palindromic sets = 6

5 paired palindromic sets = 777

10 paired palindromic sets = 3,022,530

1 paired self-palindromic sets = 3

5 paired self-palindromic sets = 777

Total pairs = 30,233,088

Total paired sets = 3,024,096

Total sets = 6,047,412

Base = 7, n = 2: Matrix is palindromic

Maximum matrix number = 48

Median = 24, median sequence = 3, 3

1 paired palindromic sets = 3

2 paired palindromic sets = 9

1 paired self-palindromic sets = 3

Total pairs = 24

Total paired sets = 15

Total sets = 27

Base = 7, n = 3: Matrix is palindromic

Maximum matrix number = 342

Median = 171, median sequence = 3, 3, 3

1 paired palindromic sets = 3

3 paired palindromic sets = 56

Total pairs = 171

Total paired sets = 59

Total sets = 118

Base = 7, n = 4: Matrix is palindromic

Maximum matrix number = 2400

Median = 1200, median sequence = 3, 3, 3, 3

1 paired palindromic sets = 3

2 paired palindromic sets = 9

4 paired palindromic sets = 288

1 paired self-palindromic sets = 3

2 paired self-palindromic sets = 12

Total pairs = 1,200

Total paired sets = 315

Total sets = 615

Base = 7, n = 5: Matrix is palindromic

Maximum matrix number = 16806

Median = 8403, median sequence = 3, 3, 3, 3, 3

1 paired palindromic sets = 3

5 paired palindromic sets = 1,680

Total pairs = 8,403

Total paired sets = 1,683

Total sets = 3,366

Base = 7, n = 6: Matrix is palindromic

Maximum matrix number = 117648

Median = 58824, median sequence = 3, 3, 3, 3, 3, 3

1 paired palindromic sets = 3

2 paired palindromic sets = 9

3 paired palindromic sets = 56

6 paired palindromic sets = 9,744

1 paired self-palindromic sets = 3

3 paired self-palindromic sets = 56

Total pairs = 58,824

Total paired sets = 9,871

Total sets = 19,683

Base = 7, n = 7: Matrix is palindromic

Maximum matrix number = 823542

Median = 411771, median sequence = 3, 3, 3, 3, 3, 3, 3

1 paired palindromic sets = 3

7 paired palindromic sets = 58,824

Total pairs = 411,771

Total paired sets = 58,827

Total sets = 117,654

Base = 7, n = 8: Matrix is palindromic

Maximum matrix number = 5764800

Median = 2882400, median sequence = 3, 3, 3, 3, 3, 3, 3, 3

1 paired palindromic sets = 3

2 paired palindromic sets = 9

4 paired palindromic sets = 288

8 paired palindromic sets = 360,000

1 paired self-palindromic sets = 3

2 paired self-palindromic sets = 12

4 paired self-palindromic sets = 300

Total pairs = 2,882,400

Total paired sets = 360,615

Total sets = 720,915

Base = 7, n = 9: Matrix is palindromic

Maximum matrix number = 40353606

Median = 20176803, median sequence = 3, 3, 3, 3, 3, 3, 3, 3, 3

1 paired palindromic sets = 3

3 paired palindromic sets = 56

9 paired palindromic sets = 2,241,848

Total pairs = 20,176,803

Total paired sets = 2,241,907

Total sets = 4,483,814

Base = 7, n = 10: Matrix is palindromic

Maximum matrix number = 282475248

Median = 141237624, median sequence = 3, 3, 3, 3, 3, 3, 3, 3, 3, 3

1 paired palindromic sets = 3

2 paired palindromic sets = 9

5 paired palindromic sets = 1,680

10 paired palindromic sets = 14,122,080

1 paired self-palindromic sets = 3

5 paired self-palindromic sets = 1,680

Total pairs = 141,237,624

Total paired sets = 14,125,455

Total sets = 28,249,227

Base = 8, n = 2: Matrix is palindromic

Maximum matrix number = 63

1 paired palindromic sets = 4

2 paired palindromic sets = 12

1 paired self-palindromic sets = 4

Total pairs = 32

Total paired sets = 20

Total sets = 36

Base = 8, n = 3: Matrix is palindromic

Maximum matrix number = 511

1 paired palindromic sets = 4

3 paired palindromic sets = 84

Total pairs = 256

Total paired sets = 88

Total sets = 176

Base = 8, n = 4: Matrix is palindromic

Maximum matrix number = 4095

1 paired palindromic sets = 4

2 paired palindromic sets = 12

4 paired palindromic sets = 496

1 paired self-palindromic sets = 4

2 paired self-palindromic sets = 16

Total pairs = 2,048

Total paired sets = 532

Total sets = 1,044

Base = 8, n = 5: Matrix is palindromic

Maximum matrix number = 32767

1 paired palindromic sets = 4

5 paired palindromic sets = 3,276

Total pairs = 16,384

Total paired sets = 3,280

Total sets = 6,560

Base = 8, n = 6: Matrix is palindromic

Maximum matrix number = 262143

1 paired palindromic sets = 4

2 paired palindromic sets = 12

3 paired palindromic sets = 84

6 paired palindromic sets = 21,756

1 paired self-palindromic sets = 4

3 paired self-palindromic sets = 84

Total pairs = 131,072

Total paired sets = 21,944

Total sets = 43,800

Base = 8, n = 7: Matrix is palindromic

Maximum matrix number = 2097151

1 paired palindromic sets = 4

7 paired palindromic sets = 149,796

Total pairs = 1,048,576

Total paired sets = 149,800

Total sets = 299,600

Base = 8, n = 8: Matrix is palindromic

Maximum matrix number = 16777215

1 paired palindromic sets = 4

2 paired palindromic sets = 12

4 paired palindromic sets = 496

8 paired palindromic sets = 1,048,064

1 paired self-palindromic sets = 4

2 paired self-palindromic sets = 16

4 paired self-palindromic sets = 512

Total pairs = 8,388,608

Total paired sets = 1,049,108

Total sets = 2,097,684

Base = 8, n = 9: Matrix is palindromic

Maximum matrix number = 134217727

1 paired palindromic sets = 4

3 paired palindromic sets = 84

9 paired palindromic sets = 7,456,512

Total pairs = 67,108,864

Total paired sets = 7,456,600

Total sets = 14,913,200

Base = 8, n = 10: Matrix is palindromic

Maximum matrix number = 1073741823

1 paired palindromic sets = 4

2 paired palindromic sets = 12

5 paired palindromic sets = 3,276

10 paired palindromic sets = 53,683,812

1 paired self-palindromic sets = 4

5 paired self-palindromic sets = 3,276

Total pairs = 536,870,912

Total paired sets = 53,690,384

Total sets = 107,377,488

Base = 9, n = 2: Matrix is palindromic

Maximum matrix number = 80

Median = 40, median sequence = 4, 4

1 paired palindromic sets = 4

2 paired palindromic sets = 16

1 paired self-palindromic sets = 4

Total pairs = 40

Total paired sets = 24

Total sets = 44

Base = 9, n = 3: Matrix is palindromic

Maximum matrix number = 728

Median = 364, median sequence = 4, 4, 4

1 paired palindromic sets = 4

3 paired palindromic sets = 120

Total pairs = 364

Total paired sets = 124

Total sets = 248

Base = 9, n = 4: Matrix is palindromic

Maximum matrix number = 6560

Median = 3280, median sequence = 4, 4, 4, 4

1 paired palindromic sets = 4

2 paired palindromic sets = 16

4 paired palindromic sets = 800

1 paired self-palindromic sets = 4

2 paired self-palindromic sets = 20

Total pairs = 3,280

Total paired sets = 844

Total sets = 1,664

Base = 9, n = 5: Matrix is palindromic

Maximum matrix number = 59048

Median = 29524, median sequence = 4, 4, 4, 4, 4

1 paired palindromic sets = 4

5 paired palindromic sets = 5,904

Total pairs = 29,524

Total paired sets = 5,908

Total sets = 11,816

Base = 9, n = 6: Matrix is palindromic

Maximum matrix number = 531440

Median = 265720, median sequence = 4, 4, 4, 4, 4, 4

1 paired palindromic sets = 4

2 paired palindromic sets = 16

3 paired palindromic sets = 120

6 paired palindromic sets = 44,160

1 paired self-palindromic sets = 4

3 paired self-palindromic sets = 120

Total pairs = 265,720

Total paired sets = 44,424

Total sets = 88,724

Base = 9, n = 7: Matrix is palindromic

Maximum matrix number = 4782968

Median = 2391484, median sequence = 4, 4, 4, 4, 4, 4, 4

1 paired palindromic sets = 4

7 paired palindromic sets = 341,640

Total pairs = 2,391,484

Total paired sets = 341,644

Total sets = 683,288

Base = 9, n = 8: Matrix is palindromic

Maximum matrix number = 43046720

Median = 21523360, median sequence = 4, 4, 4, 4, 4, 4, 4, 4

1 paired palindromic sets = 4

2 paired palindromic sets = 16

4 paired palindromic sets = 800

8 paired palindromic sets = 2,689,600

1 paired self-palindromic sets = 4

2 paired self-palindromic sets = 20

4 paired self-palindromic sets = 820

Total pairs = 21,523,360

Total paired sets = 2,691,264

Total sets = 5,381,684

Base = 9, n = 9: Matrix is palindromic

Maximum matrix number = 387420488

Median = 193710244, median sequence = 4, 4, 4, 4, 4, 4, 4, 4, 4

1 paired palindromic sets = 4

3 paired palindromic sets = 120

9 paired palindromic sets = 21,523,320

Total pairs = 193,710,244

Total paired sets = 21,523,444

Total sets = 43,046,888

Base = 9, n = 10: Matrix is palindromic

Maximum matrix number = 3486784400

Median = 1743392200, median sequence = 4, 4, 4, 4, 4, 4, 4, 4, 4, 4

1 paired palindromic sets = 4

2 paired palindromic sets = 16

5 paired palindromic sets = 5,904

10 paired palindromic sets = 174,333,312

1 paired self-palindromic sets = 4

5 paired self-palindromic sets = 5,904

Total pairs = 1,743,392,200

Total paired sets = 174,345,144

Total sets = 348,684,380

Base = 10, n = 2: Matrix is palindromic

Maximum matrix number = 99

1 paired palindromic sets = 5

2 paired palindromic sets = 20

1 paired self-palindromic sets = 5

Total pairs = 50

Total paired sets = 30

Total sets = 55

Base = 10, n = 3: Matrix is palindromic

Maximum matrix number = 999

1 paired palindromic sets = 5

3 paired palindromic sets = 165

Total pairs = 500

Total paired sets = 170

Total sets = 340

Base = 10, n = 4: Matrix is palindromic

Maximum matrix number = 9999

1 paired palindromic sets = 5

2 paired palindromic sets = 20

4 paired palindromic sets = 1,225

1 paired self-palindromic sets = 5

2 paired self-palindromic sets = 25

Total pairs = 5,000

Total paired sets = 1,280

Total sets = 2,530

Base = 10, n = 5: Matrix is palindromic

Maximum matrix number = 99999

1 paired palindromic sets = 5

5 paired palindromic sets = 9,999

Total pairs = 50,000

Total paired sets = 10,004

Total sets = 20,008

Base = 10, n = 6: Matrix is palindromic

Maximum matrix number = 999999

1 paired palindromic sets = 5

2 paired palindromic sets = 20

3 paired palindromic sets = 165

6 paired palindromic sets = 83,160

1 paired self-palindromic sets = 5

3 paired self-palindromic sets = 165

Total pairs = 500,000

Total paired sets = 83,520

Total sets = 166,870

Base = 10, n = 7: Matrix is palindromic

Maximum matrix number = 9999999

1 paired palindromic sets = 5

7 paired palindromic sets = 714,285

Total pairs = 5,000,000

Total paired sets = 714,290

Total sets = 1,428,580

Base = 10, n = 8: Matrix is palindromic

Maximum matrix number = 99999999

1 paired palindromic sets = 5

2 paired palindromic sets = 20

4 paired palindromic sets = 1,225

8 paired palindromic sets = 6,248,750

1 paired self-palindromic sets = 5

2 paired self-palindromic sets = 25

4 paired self-palindromic sets = 1,250

Total pairs = 50,000,000

Total paired sets = 6,251,280

Total sets = 12,501,280

Base = 10, n = 9: Matrix is palindromic

Maximum matrix number = 999999999

1 paired palindromic sets = 5

3 paired palindromic sets = 165

9 paired palindromic sets = 55,555,500

Total pairs = 500,000,000

Total paired sets = 55,555,670

Total sets = 111,111,340

Base = 10, n = 10: Matrix is palindromic

Maximum matrix number = 9999999999

1 paired palindromic sets = 5

2 paired palindromic sets = 20

5 paired palindromic sets = 9,999

10 paired palindromic sets = 499,989,996

1 paired self-palindromic sets = 5

5 paired self-palindromic sets = 9,999

Total pairs = 5,000,000,000

Total paired sets = 500,010,024

Total sets = 1,000,010,044

Processing finished successfully

All matrices were palindromic

See above for details

Total process time (hh:mm:ss.sss) = 00:50:17.568

**Appendix B – Computational results for permutations only**

Checking palindromicity for permutations only for Base = 2 to 10

### Permutations for Base = 2 ###

Base = 2, n = 2: Matrix is palindromic

Maximum matrix number = 3

1 paired self-palindromic sets = 1

Total pairs = 1

Total paired sets = 1

Total sets = 1

### Permutations for Base = 3 ###

Base = 3, n = 3: Matrix is palindromic

Maximum matrix number = 26

Median = 13, median sequence = 1, 1, 1

3 paired palindromic sets = 1

Total pairs = 3

Total paired sets = 1

Total sets = 2

### Permutations for Base = 4 ###

Base = 4, n = 4: Matrix is palindromic

Maximum matrix number = 255

4 paired palindromic sets = 2

2 paired self-palindromic sets = 2

Total pairs = 12

Total paired sets = 4

Total sets = 6

### Permutations for Base = 5 ###

Base = 5, n = 5: Matrix is palindromic

Maximum matrix number = 3124

Median = 1562, median sequence = 2, 2, 2, 2, 2

5 paired palindromic sets = 12

Total pairs = 60

Total paired sets = 12

Total sets = 24

### Permutations for Base = 6 ###

Base = 6, n = 6: Matrix is palindromic

Maximum matrix number = 46655

6 paired palindromic sets = 56

3 paired self-palindromic sets = 8

Total pairs = 360

Total paired sets = 64

Total sets = 120

### Permutations for Base = 7 ###

Base = 7, n = 7: Matrix is palindromic

Maximum matrix number = 823542

Median = 411771, median sequence = 3, 3, 3, 3, 3, 3, 3

7 paired palindromic sets = 360

Total pairs = 2,520

Total paired sets = 360

Total sets = 720

### Permutations for Base = 8 ###

Base = 8, n = 8: Matrix is palindromic

Maximum matrix number = 16777215

8 paired palindromic sets = 2,496

4 paired self-palindromic sets = 48

Total pairs = 20,160

Total paired sets = 2,544

Total sets = 5,040

### Permutations for Base = 9 ###

Base = 9, n = 9: Matrix is palindromic

Maximum matrix number = 387420488

Median = 193710244, median sequence = 4, 4, 4, 4, 4, 4, 4, 4, 4

9 paired palindromic sets = 20,160

Total pairs = 181,440

Total paired sets = 20,160

Total sets = 40,320

### Permutations for Base = 10 ###

Base = 10, n = 10: Matrix is palindromic

Maximum matrix number = 9999999999

10 paired palindromic sets = 181,248

5 paired self-palindromic sets = 384

Total pairs = 1,814,400

Total paired sets = 181,632

Total sets = 362,880

Processing finished successfully

All matrices were palindromic

See above for details

Total process time (hh:mm:ss.sss) = 00:10:32.123